

Debt-To-Pay-Debt Lemma: Lessons from Japan

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Abstract

I propose and empirically test Debt-To-Pay-Debt Lemma, which posits that a significant portion of an entity's current debt is allocated to servicing its previous period's debt obligations. Japan, despite being categorized as a wealthy nation, ranks among the highest in public-debt globally. I test the lemma using autoregressive integrated moving average (ARIMA) modeling technique. I utilize World Bank quarterly time-series from 1990 to 2022. Dependent variable is Japan's central government debt, total (% of GDP), while moving average (MA) and autoregressive (AR) components function as independent variables. Conditional least squares parameter estimate yields a statistically significant AR(1) coefficient of 0.75534, indicating that approximately 76% of Japan's current-year debt is allocated to servicing previous-year debt obligations, thereby empirically validating Debt-To-Pay-Debt Lemma. ARIMA(1,1,1) diagnostics yield an adjusted R-squared of 0.652144, suggesting that the model explains 65.2% of public-debt variations. These findings underscore a self-perpetuating nature of public-debt accumulation in Japan, challenging conventional perceptions of a looming debt crisis. I recommend re-thinking conservative fiscal policy frameworks by adopting adaptive, context-sensitive strategies that account for cyclical interplay between borrowing and debt servicing.

Introduction

Public-debt has long been a subject of economic discourse, particularly in the context of its sustainability and implications for macroeconomic stability. While high debt levels are often associated with financial distress, Japan presents a paradox: it has one of the highest public-debt-to-GDP ratios globally, yet it remains a wealthy and economically stable nation (Krugman, 1998). This phenomenon challenges conventional debt theories and raises critical questions about the mechanics of public-debt accumulation and servicing. I propose and empirically test Debt-To-Pay-Debt Lemma, which posits that a substantial portion of an entity's current debt is allocated to servicing previous period obligations. Unlike developing economies where high debt levels often lead to fiscal crises, Japan has managed to sustain its economic growth despite its debt burden (Blanchard, 2019). This paradox calls for an empirical test on whether Japan's public-debt is inherently self-perpetuating, where new borrowing primarily finances past liabilities rather than productive investments. I test the lemma using autoregressive integrated moving average (ARIMA) modeling technique on Japan's public-debt data from 1990 to 2022. Dependent variable is central government debt, total (% of GDP). Moving average (MA) and autoregressive (AR) components are the independent variables (Nahabwe & Kagarura, 2025). Conditional least squares (CLS) estimation technique is applied to test the extent to which Japan's current debt obligations are influenced by past liabilities. The rationale for this study is threefold. First, it contributes to the theoretical discourse on public-debt dynamics, particularly regarding the sustainability of high debt levels in advanced economies. Second, it provides empirical validation of the Debt-To-Pay-Debt Lemma, offering insights for policymakers on debt management strategies. Third, understanding Japan's debt structure offers lessons for other nations grappling with rising debt burdens,



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especially in an era of heightened fiscal pressures and global economic uncertainty (Reinhart & Rogoff, 2011). The study is significant for both academic and policy-making communities. If Debt-To-Pay-Debt Lemma holds, it challenges the conventional assumption that fiscal austerity alone can restore debt sustainability. Instead, it points toward the necessity of a more nuanced and dynamic approach that acknowledges the cyclical nature of debt servicing and borrowing requirements (Bohn, 1995; Reinhart & Rogoff, 2010). Traditional models often overlook the recursive feedback between existing debt and new borrowing, thereby underestimating the structural inertia in public-debt systems (Ostry et al., 2015). By employing ARIMA modeling, the research captures persistence and autoregressive patterns in Japan's debt trajectory, revealing endogenous mechanisms that sustain fiscal stability despite historically unprecedented levels of public debt (Enders, 2014; Hoshi & Kashyap, 2015).

Literature Review

Accumulation of public-debt is a persistent theme in literature; scholars often debate its sustainability and implications for macroeconomic stability. Keynesian economics argues that public-debt, when used effectively, can stimulate economic growth by financing productive investments (Keynes, 1936). However, Ricardian equivalence hypothesis contends that debt-financed expenditures merely shift tax burden to future generations, limiting their effectiveness (Barro, 1974). Japan's case challenges these traditional views, as it maintains economic stability despite a public-debt exceeding 200% of GDP (IMF, 2022). Empirical studies highlight that Japan's debt is largely domestically owned, mitigating risks of external vulnerabilities (Krugman, 1998). Recent studies on developed economies reveal that sustained public-debt can lead to structural stagnation if not accompanied by productivity gains (Blanchard, 2019). Conversely, Modern Monetary Theory (MMT) suggests that sovereign nations with control over their currency can sustain high debt levels indefinitely without risking default, provided inflation remains controlled (Contessa, 2020). These debates inform Debt-To-Pay-Debt Lemma, which posits that a substantial portion of public-debt is used for debt servicing rather than economic expansion. In the broader Asian context, public-debt sustainability remains a significant policy concern. Studies on China indicate that although debt-financed infrastructure investments drive growth, excessive reliance on borrowing risks financial instability (Lardy, 2019). In contrast, India's fiscal policies emphasize balancing debt accumulation with structural reforms to sustain economic growth (Mohanty, 2020). Southeast Asian economies, such as Indonesia and Malaysia, have adopted debt management strategies involving a mix of domestic borrowing and foreign capital to maintain macroeconomic stability (ADB, 2021).

Japan's unique debt structure characterized by low external borrowing and high domestic ownership sets it apart from other Asian economies. While developing nations struggle with debt servicing due to high interest rates and reliance on external creditors, Japan benefits from low borrowing costs, allowing for continued fiscal expansion without triggering immediate financial distress (Reinhart & Rogoff, 2011). This raises critical questions on whether Japan's model can be replicated in other regions facing rising Public-debt levels. In Africa and Latin America, high public-debt levels have often led to fiscal crises due to external borrowing and unfavorable debt servicing conditions. Studies on Sub-Saharan Africa reveal that excessive debt accumulation, particularly from international lenders, results in prolonged economic instability and reliance on structural adjustment programs (Battaile, 2015). Uganda and Kenya, for example, have witnessed rising debt-to-GDP ratios, prompting debates on the long-term sustainability of their fiscal policies (Nkatha, 2022). Japan's experience offers insights into debt

sustainability strategies for developing economies. The emphasis on domestic borrowing and low-interest debt structures could serve as a model for countries struggling with external debt burdens. However, differences in financial market maturity and monetary policy autonomy limit the direct applicability of Japan's debt approach in these regions (Iyoha, 1999). Debt-To-Pay-Debt Lemma is grounded in public-debt theories, including Debt Overhang Theory, which suggests that excessive debt accumulation reduces economic growth by diverting resources from productive investments (Krugman, 1988). Additionally, Intertemporal Budget Constraint Theory argues that a government's long-term fiscal position must be sustainable to prevent debt crises (Blanchard & Fischer, 1989). These theories provide a basis for understanding the extent to which current debt obligations are met using new borrowings, a core tenet of the proposed lemma. The conceptual framework is structured around Japan's public-debt servicing mechanisms. The dependent variable is central government debt, total (% of GDP). Moving average (MA) and autoregressive (AR) components function as independent variables (Nahabwe & Kagarura, 2025). Debt-To-Pay-Debt Lemma hypothesizes that Japan's high debt levels are primarily sustained through a cycle of continuous borrowing to meet past obligations rather than financing new economic activities. The study employs ARIMA model to empirically test this hypothesis, with conditional least squares (CLS) estimation providing statistical validation.

Data And Methods

I utilize a quantitative research design to empirically test Debt-To-Pay-Debt Lemma using Japan's public-debt data from 1990 to 2022. I employ time-series econometric approach, specifically leveraging ARIMA model to analyze the dynamics of debt servicing. Selection of ARIMA modeling is justified by its robustness in capturing trends, cycles, and stochastic processes in time-series data (Box & Jenkins, 1976; Nahabwe & Kagarura, 2025). I use secondary data obtained from the World Bank database, covering the period 1990-2022. The dataset includes annual observations on Japan's central government debt, total (% of GDP), ensuring a comprehensive analysis of debt trends over time. However, to increase the degrees of freedom and provide a more granular analysis, I transform the data into quarterly time-series, a process that allows for higher-frequency analysis and better statistical inference (Gujarati & Porter, 2009; Kagarura & Nahabwe, 2025). Transformation ensures a more detailed examination of quarterly fluctuations in debt trends and better captures seasonal or cyclical variations. Sample consists of 128 quarterly data points (33 years \times 4 quarters per year). This approach allows for the examination of quarterly dynamics while maintaining the integrity of the data across a broad time horizon (Kagarura & Nahabwe, 2025). I adopt a purposive sampling approach given that the study's focus is on national-level, selecting Japan as a case study due to its uniquely high public-debt levels despite economic stability (IMF, 2022). Use of a single-country analysis allows for in-depth exploration of the proposed lemma while controlling for country-specific fiscal policies and economic conditions (Krugman, 2009). ARIMA modeling, a widely accepted time-series forecasting technique, is used to assess the extent to which Japan's current debt obligations are used to service previous debt. The general form of an ARIMA(p,d,q) model is expressed as:

$$Y_t = c + \sum_{i=1}^p \phi_i Y_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t \dots\dots\dots (1)$$

Where;

Y_t is Central government debt, total (% of GDP) at time t

c is constant term

ε_t is white noise at time t

ϕ_i are the coefficients of the autoregressive terms

θ_j are the coefficients of the moving average terms

p = Number of lagged AR terms

d = Number of differences required to make the series stationary

q = Number of lagged MA terms (Box & Jenkins 1976; Nahabwe & Maniple, 2025)

I estimate parameters using conditional least squares (CLS) method, employing Gauss-Newton/Marquardt steps for optimization. CLS method minimizes the sum of squared residuals and provides efficient parameter estimates, particularly in the presence of autocorrelation and heteroskedasticity (Hamilton, 1994; Nahabwe & Maniple, 2025). The Gauss-Newton/Marquardt algorithm is suitable for non-linear optimization problems, ensuring convergence to a local minimum and improving estimation accuracy (Marquardt, 1963; Nahabwe & Kagarura, 2025). CLS estimator for the regression coefficients is given by the following formula:

$$\hat{\theta} = \operatorname{argmin}_{\theta} [\sum_{t=1}^n (y_t - \hat{y}_t(\theta))^2] \dots\dots\dots (2)$$

Where:

$\hat{\theta}$ represents the estimated parameter vector (which includes both AR and MA parameters in ARIMA).

y_t represents the actual observed value of the dependent variable at time t

$\hat{y}_t(\theta)$ represents the model's predicted value at time t based on the parameter estimates θ

n is the number of observations. (Hamilton, 1994; Nahabwe & Kagarura, 2025).

I focus on a longitudinal dataset (1990 to 2022) to capture historical debt patterns and trends, allowing for a robust evaluation of debt servicing mechanisms (Enders, 2014; Kagarura & Nahabwe, 2025). I chose ARIMA approach due to its ability to model stochastic processes, making it ideal for forecasting public-debt accumulation and its structural dependencies (Gujarati & Porter, 2009; Nahabwe & Kagarura, 2025). CLS is preferred over ordinary least squares (OLS) due to the autocorrelated nature of time-series data, ensuring unbiased and efficient parameter estimates (Lütkepohl, 2005; Nahabwe & Maniple, 2025).

Results And Their Discussion

In this section I present both descriptive and inferential statistical results guided by the study objectives, particularly to assess the persistence of Japan's central government debt and validate Debt-To-Pay-Debt Lemma using quarterly data from 1990 to 2022. Mean debt-to-GDP ratio is 132.32%, with a median of 128.60%, indicating persistently high debt levels throughout the study period. The peak of 216.33% occurred in 2021 amid intensified fiscal pressures, while the lowest value of 38.18% was recorded in 1991, prior to the acceleration in debt accumulation (IMF, 2022). The standard deviation of 57.63% suggests substantial fluctuations in debt levels, which aligns with Japan's periods of economic contraction and expansive fiscal policy (Krugman, 1998). Stationarity test using Augmented Dickey-Fuller shows original debt series is non-stationary ($p > 0.05$) but achieves stationarity upon first differencing ($p < 0.05$). Consequently, an ARIMA(1,1,1) model is identified as the best fit based on model selection criteria (AIC=2.57; SC=2.64) (Nahabwe & Kagarura, 2025).

The model output is estimated as:
 Results of ARIMA(1,1,1) model (Appendix 4)

$$\widehat{Debt}_t = 1.399947 + 0.755340AR(1) + 0.084593 MA(1) \dots\dots\dots (3)$$

Hence,

$$\hat{\theta}_{CLS} = \begin{bmatrix} 1.399947 \\ 0.755340 \\ 0.084593 \end{bmatrix}$$

Constant term ($C = 1.40$, $p = 0.0001$) is statistically significant and positive, highlighting a baseline upward trend in debt accumulation, even in the absence of autoregressive or moving average influences. This finding suggests an inherent structural pressure toward increasing debt, largely driven by long-term fiscal expansion and demographic factors (Blanchard, 2021; Nahabwe & Kagarura, 2025). AR(1) coefficient (0.76, $p = 0.0000$) is also highly significant, confirming that past debt levels strongly determine current borrowing behavior. This supports Debt-To-Pay-Debt Lemma, wherein nearly 76% of current debt is explained by prior debt servicing needs. The result echoes prior findings by Reinhart & Rogoff (2010), who emphasized self-reinforcing debt cycles in economies with entrenched borrowing patterns. Conversely, the MA(1) coefficient (0.08, $p = 0.4362$) is statistically insignificant, indicating that short-term shocks or fluctuations exert negligible influence on long-run debt trends. This implies that Japan's debt accumulation is structurally driven rather than reactive to temporary disturbances, which aligns with Arestis (2005), who highlights limitations of cyclical interventions in high-debt economies. Adjusted R-squared of 0.652 implies that the ARIMA model explains over 65% of the variance in Japan's debt trajectory. The Durbin-Watson statistic (1.98) is close to 2, suggesting low residual autocorrelation and enhancing model credibility (Wooldridge, 2019; Nahabwe & Kagarura, 2025). Although the residuals are not normally distributed (Jarque-Bera = 351.01, $p = 0.000$), the Ljung-Box Q statistic ($p = 0.543$) confirms that the residuals behave like white noise (Nahabwe & Kagarura, 2025). This suggests no remaining systematic pattern in the errors, which is critical for validating the model's reliability (Box et al., 2016; Nahabwe & Kagarura, 2025). Furthermore, the AR and MA roots lie within the unit circle (Appendix 6), confirming model stability, stationarity, and invertibility essential features for accurate forecasting (Hamilton, 1994; Munyambonera et al., 2025). Model simulations (Appendix 8) extending to 2060 forecast a structural increase in debt-to-GDP, driven primarily by the significant autoregressive component. The dominance of AR(1) in shaping future debt dynamics supports the view that Japan's debt system is path-dependent, with current obligations largely determined by historical debt accumulation. The study makes a unique contribution by proposing and empirically validating Debt-To-Pay-Debt Lemma within a robust ARIMA(1,1,1) framework, emphasizing that Japan's debt trajectory is not simply a byproduct of policy shocks but a systemic outcome of long-term fiscal structures. The finding challenges conventional debt sustainability narratives and calls for a fundamental re-thinking of fiscal policy discipline in aging, high-debt economies.

Limitations

Despite analytical strength of ARIMA (1,1,1) model employed, I acknowledge several methodological constraints. The model is fundamentally linear and, therefore, may not adequately capture non-linearities inherent in Japan's complex debt dynamics such as abrupt fiscal policy changes, macroeconomic shocks, or geopolitical disruptions (Enders, 2014; Nahabwe & Kagarura, 2025). Conditional least squares (CLS) estimation technique, although suitable for ARIMA models, is particularly sensitive to structural breaks and outliers, which may bias coefficient estimates in the presence of irregular economic events

(Stock & Watson, 2020; Nahabwe & Kagarura, 2025). Furthermore, I did not explicitly account for the impact of unconventional monetary policy measures such as Bank of Japan's yield curve control or persistent negative interest rate policy factors that significantly influence debt trajectory (Doi et al., 2010). The study's temporal scope, relying on quarterly World Bank data from 1990 to 2022, offers valuable insights but omits crucial historical contexts, including Japan's post-war fiscal restructuring and aftermath of 1980s asset price bubble, which could provide a more holistic view of long-term debt cycles (Krugman, 1998). Additionally, the data, though from a reputable source, may contain revisions and potential measurement errors that can affect the robustness of parameter estimates (Croushore, 2011). The findings are also specific to Japan's distinctive economic framework marked by high domestic savings, an aging population, and a captive bond market limiting direct applicability of the results to countries with different fiscal and institutional arrangements (Hoshi & Kashyap, 2015). Future studies could consider broader econometric approaches, such as Structural VAR or Markov-Switching models, and longer, real-time datasets to better capture structural shifts and externalities in sovereign debt evolution.

Conclusion

I advance the understanding of sovereign debt dynamics by unveiling the structural inertia embedded within Japan's public-debt trajectory. Beyond merely quantifying debt levels, I expose a deeper institutional pattern: a recursive borrowing mechanism wherein new debt issuance is largely committed to repaying existing obligations (Doi et al., 2010). Empirical findings, grounded in a robust ARIMA(1,1,1) estimation, underscore systemic entrenchment of "Debt-To-Pay-Debt Lemma," a framework that transcends conventional fiscal interpretations and calls attention to compounding nature of debt dependency (Lee et al., 2016). In this context, Japan's fiscal architecture appears less responsive to traditional corrective measures, revealing a policy loop that sustains high debt levels irrespective of growth-oriented or austerity-driven interventions (Reinhart & Rogoff, 2011; Bohn, 1995). This structural insight has profound implications for both economic theory and policymaking. It challenges orthodox prescriptions that emphasize short-term budget tightening and instead advocates for comprehensive debt management strategies that internalize cyclical feedback between debt servicing and borrowing (Eyraud & Wu, 2015; Yoshino et al, 2017). Addressing such entrenched debt dynamics requires re-thinking fiscal sustainability through a lens that incorporates long-term institutional and demographic constraints, as well as macroeconomic feedback loops. As Japan's case illustrates, sustainability cannot merely be enforced through austerity, but must be structurally engineered to prevent the perpetuation of debt-financed debt, a lesson with global significance for high-debt economies.

Recommendations

Recognizing the cyclical and autoregressive nature of Japan's public debt, it is imperative for policymakers to reframe debt management beyond conventional austerity. I confirm that new borrowing is substantially driven by past obligations, suggesting that fiscal policy must internalize this inherent cyclicality (Doi et al., 2010). Japan should adopt a dynamic fiscal rule, one that adjusts spending and borrowing thresholds based on the historical burden of debt servicing (Eyraud & Wu, 2015). Instead of aggressive cuts, a gradual reallocation of fiscal space toward productive investments particularly in innovation, green infrastructure, and demographic resilience can stimulate economic growth while softening the self-perpetuating debt cycle (Kuttner & Posen, 2002; Furceri et al., 2021). Equally vital is strengthening automatic stabilizers, such as counter-cyclical tax policies and flexible public investment programs, which adapt with the debt cycle and mitigate

future reliance on debt-funded debt repayments (OECD, 2021). On programmatic and research fronts, Japan should institutionalize a cyclicity-aware debt restructuring framework, one that not only adjusts maturity and interest profiles based on debt servicing patterns but also integrates real-time economic signals into refinancing strategies (Yoshino et al, 2017). Programmatically, public communication on debt sustainability should be broadened to improve fiscal literacy and public support for cyclical debt-sensitive reforms (Frieden, 1992). In terms of research, future work should deepen understanding of the structural roots of this cyclicity such as demographic aging, intergenerational fiscal transfers, and health expenditures and their feedback loops into borrowing behavior (Lee et al., 2016). Moreover, applying alternative modeling techniques like state-space models or Markov-switching frameworks could reveal how shifts in economic regimes interact with debt cycles, offering more responsive policy pathways in managing Japan's entrenched debt dynamics (Bohn, 1995; Reinhart & Rogoff, 2011).

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Appendices

Appendix 1: Descriptive statistics

	Central government debt, total (% of GDP)
Mean	132.3151
Median	128.5997
Maximum	216.3294
Minimum	38.17619
Std. Dev.	57.62541
Skewness	-0.146734
Kurtosis	1.682039
Jarque-Bera	9.799401
Probability	0.007449
Sum	17068.65
Sum Sq. Dev.	425048.1
Observations	129

Appendix 2: Unit root test, debt (in Level)

Null Hypothesis: D(DEBT) has a unit root

Exogenous: Constant

Lag Length: 4 (Automatic - based on SIC, maxlag=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.308556	0.0166
Test critical values:		
1% level	-3.484198	
5% level	-2.885051	
10% level	-2.579386	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(DEBT,2)

Method: Least Squares

Date: 03/30/25 Time: 21:00

Sample (adjusted): 1991Q3 2022Q1

Included observations: 123 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.

D(DEBT(-1))	-0.222610	0.067283	-3.308556	0.0012
D(DEBT(-1),2)	0.088587	0.078859	1.123354	0.2636
D(DEBT(-2),2)	0.088587	0.078859	1.123354	0.2636
D(DEBT(-3),2)	0.088587	0.078859	1.123354	0.2636
D(DEBT(-4),2)	-0.328588	0.078822	-4.168723	0.0001
C	0.322747	0.117119	2.755716	0.0068
R-squared	0.266802	Mean dependent var	-0.001737	
Adjusted R-squared	0.235469	S.D. dependent var	0.878115	
S.E. of regression	0.767802	Akaike info criterion	2.356980	
Sum squared resid	68.97374	Schwarz criterion	2.494159	
Log likelihood	-138.9542	Hannan-Quinn criter.	2.412702	
F-statistic	8.514989	Durbin-Watson stat	1.915599	
Prob(F-statistic)	0.000001			

Appendix 3: Unit root test, debt (in First difference)

Null Hypothesis: DEBT has a unit root

Exogenous: Constant

Lag Length: 5 (Automatic - based on SIC, maxlag=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.554587	0.5027
Test critical values: 1% level	-3.484198	
5% level	-2.885051	
10% level	-2.579386	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(DEBT)

Method: Least Squares

Date: 03/30/25 Time: 21:01

Sample (adjusted): 1991Q3 2022Q1

Included observations: 123 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DEBT(-1)	-0.001973	0.001269	-1.554587	0.1228
D(DEBT(-1))	0.851129	0.084469	10.07618	0.0000
D(DEBT(-2))	0.000876	0.104173	0.008413	0.9933
D(DEBT(-3))	0.000876	0.104173	0.008413	0.9933
D(DEBT(-4))	-0.416051	0.104181	-3.993528	0.0001
D(DEBT(-5))	0.338745	0.078621	4.308564	0.0000
C	0.593343	0.209406	2.833463	0.0054
R-squared	0.660271	Mean dependent var	1.445955	

Adjusted R-squared	0.642698	S.D. dependent var	1.276785
S.E. of regression	0.763195	Akaike info criterion	2.352620
Sum squared resid	67.56607	Schwarz criterion	2.512663
Log likelihood	-137.6861	Hannan-Quinn criter.	2.417629
F-statistic	37.57471	Durbin-Watson stat	1.921832
Prob(F-statistic)	0.000000		

Appendix 4: Results of ARIMA(1,1,1) model

Dependent Variable: D(DEBT)

Method: ARMA Conditional Least Squares (Gauss-Newton / Marquardt steps)

Date: 03/30/25 Time: 21:04

Sample (adjusted): 1990Q3 2022Q1

Included observations: 127 after adjustments

Failure to improve likelihood (non-zero gradients) after 7 iterations

Coefficient covariance computed using outer product of gradients

MA Backcast: 1990Q2

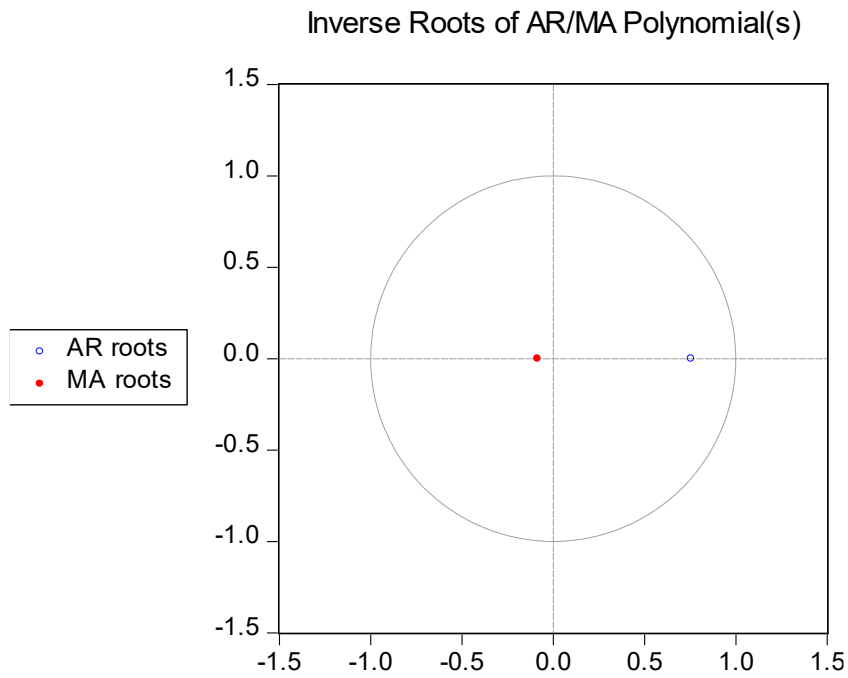
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.399947	0.341131	4.103843	0.0001
AR(1)	0.755340	0.065222	11.58114	0.0000
MA(1)	0.084593	0.108293	0.781152	0.4362
R-squared	0.657665	Mean dependent var		1.320156
Adjusted R-squared	0.652144	S.D. dependent var		1.465591
S.E. of regression	0.864396	Akaike info criterion		2.569766
Sum squared resid	92.65027	Schwarz criterion		2.636952
Log likelihood	-160.1801	Hannan-Quinn criter.		2.597063
F-statistic	119.1094	Durbin-Watson stat		1.981595
Prob(F-statistic)	0.000000			
Inverted AR Roots	.76			
Inverted MA Roots	-.08			

Appendix 5: Ljung-Box Q statistic/ test

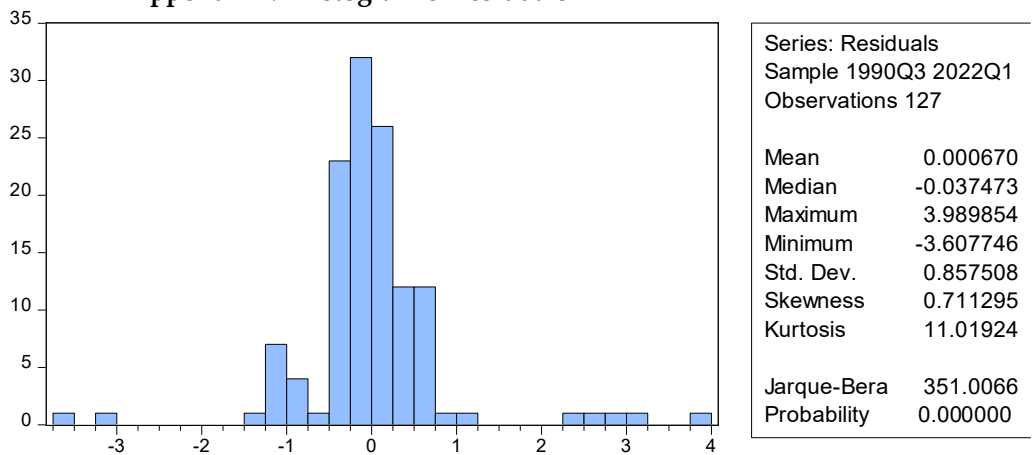
Date: 03/30/25 Time: 21:16
 Sample: 1990Q1 2022Q4
 Included observations: 127
 Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.001	0.001	0.0002	
		2	0.043	0.043	0.2474	
		3	0.030	0.030	0.3691	0.543
		4	-0.361	-0.364	17.691	0.000
		5	0.035	0.041	17.851	0.000
		6	-0.000	0.040	17.851	0.001
		7	-0.012	0.005	17.872	0.003
		8	0.138	0.003	20.499	0.002
		9	-0.026	-0.003	20.589	0.004
		10	-0.019	-0.018	20.640	0.008
		11	-0.026	-0.036	20.736	0.014
		12	-0.040	0.019	20.961	0.021
		13	0.007	-0.003	20.969	0.034
		14	0.002	-0.014	20.970	0.051
		15	0.016	-0.004	21.005	0.073
		16	-0.151	-0.183	24.387	0.041
		17	-0.005	0.007	24.391	0.059
		18	-0.013	0.011	24.414	0.081
		19	-0.013	0.007	24.439	0.108
		20	0.051	-0.077	24.842	0.129
		21	-0.011	-0.002	24.860	0.165
		22	-0.006	-0.004	24.866	0.207
		23	-0.016	-0.026	24.904	0.251
		24	0.077	0.122	25.836	0.259
		25	-0.012	-0.027	25.857	0.308
		26	-0.010	-0.041	25.873	0.360
		27	-0.006	-0.038	25.879	0.414
		28	-0.118	-0.064	28.191	0.349
		29	-0.011	-0.023	28.211	0.400
		30	-0.018	-0.022	28.266	0.450
		31	-0.007	-0.000	28.274	0.503
		32	-0.101	-0.263	30.024	0.464
		33	0.013	0.027	30.052	0.515
		34	0.017	0.043	30.104	0.563
		35	0.005	0.013	30.109	0.612
		36	0.298	0.272	46.065	0.081

Appendix 6: ARIMA(1,1,1) structure



Appendix 7: Histogram of residuals



Appendix 8: Graph showing Japan's Debt: ARIMA(1,1,1) Model Simulation

